Atomic Spectroscopy of a Hydrogen Spectrum

Taylor Larrechea, Colorado Mesa University

April 19, 2018

Abstract

We aim to find the wavelengths of light produced by hydrogen as the source of the light as predicted by quantum theory. A spectroscope was used in this experiment to display the spectrum of light that was to be observed. The colors of light observed were red, green, and purple. Red was found to have a wavelength of 675.9 ± 1.8 nm where green had a wavelength of 498.9 ± 0.1 nm and purple had a wavelength of 445.3 ± 1.1 nm. The theoretical wavelength value for red light is 656.1 nm where as green and purple are 486.0 nm and 433.9 nm respectively.

1 Background

We aim to address whether we can predict the wavelengths of visible light emitted by Hydrogen. Quantum theory can predict the wavelengths of emitted light and we aim to see if we can get close to these predicted values. The mechanics of this experiment were carried out with the use of a spectroscope and a diffraction grating to observe a hydrogen source. The first two maxima (Where $m \pm 1, 2$) were the brightest maxima and gave the best experimental data compared to the theoretical values. The wavelengths of material are useful because each element has it's own distinguished wavelength of light. In essence, if someone were to observe something and only be able to know the wavelength they could still identify it with the use of spectroscopy. By carrying out this experiment we are also checking the validity of Quantum Theory.

2 Theory

The energy of a photon is [3]

$$E_{Photon} = hf, \tag{1}$$

where h is Planck's constant and f is the frequency of the light from the photon. Knowing that the speed of light is related to $c = \lambda f$ [3] equation (1) can be manipulated to solving for the wavelength of light λ giving

$$\lambda = \frac{hc}{E_{Photon}}. \tag{2}$$

Equation (2) is useful for calculating the wavelength of light only when the energy of the photon is known. The relation of the energy of a photon to the energy of an atom is [3]

$$-\triangle E_{Atom} = E_{Photon}, \tag{3}$$

where the $\triangle E_{Atom}$ in equation (3) is the change in energy of an atom when changing between energy levels. Each energy level corresponds to a certain principle quantum number as well. Since the element being used in this experiment was Hydrogen we are particularly interested in the Hydrogen atom energy levels. Starting off at the ground state where $n = 0$, the energy levels for the Hydrogen atom are; $13.6, 3.40, 1.51, 0.85, 0.54, 0.38$ [3] all in units of electron volts (eV). Knowing the value for E_0 as well as the principle quantum numbers we can calculate the theoretical value for the wavelength of light. The equation for calculating the theoretical wavelength of light is [3]

$$\frac{1}{\lambda} = \frac{E_0}{hc}[\frac{1}{n_f^2} - \frac{1}{n_i^2}], \tag{4}$$

where n_f and n_i correspond to final and initial principal quantum numbers in this experiment. The variable energy level E_0 can be calculated by $E_0 = \frac{mk^2e^4}{4\pi c\hbar^3}$. For the energy level E_0, m is the mass and e is the charge of an electron, c is the speed of light, $\hbar$ is $\frac{h}{2\pi}$, and k is Coulomb's constant. Making this substitution for E_0 equation (4) becomes

$$\frac{1}{\lambda} = \frac{mk^2e^4}{4\pi c\hbar^3}[\frac{1}{n_f^2} - \frac{1}{n_i^2}] \tag{5}$$

where equation (5) gives us the wavelength in terms of parameters from this experiment. An algebraic substitution in equation (5) can be made with $R_\infty = \frac{mk^2e^4}{4\pi c\hbar^3}$ thus yielding the Rydberg equation. The Rydberg equation is thus [3]

$$\frac{1}{\lambda} = R_\infty[\frac{1}{n_f^2} - \frac{1}{n_i^2}], \tag{6}$$

giving a way to calculate the theoretical wavelength of light for this experiment. Knowing the principle quantum numbers will allow us to calculate the wavelengths of light from this experiment. It should be noted that for this experiment n_f was always equal to 2, where as the n_i value ranged from 3 to 5. Once the wavelengths of light are found they can be used to approximate the value of E_0. The equation for change of energy according to principal quantum number states is [3]

$$\triangle E = -E_0[\frac{1}{n_f^2} - \frac{1}{n_i^2}], \tag{7}$$

where equation (7) looks very similar to the Rydberg Equation. $\triangle E$ can also be stated as $\triangle E = hf$ where h is Planck's constant and f is the frequency light. Using this relationship as well as $c = \lambda f$ equation (7) becomes

$$E_0 = -\frac{hc}{\lambda}[\frac{1}{n_f^2} - \frac{1}{n_i^2}]^{-1}, \tag{8}$$

where λ is the wavelength of light and the other variables are the same from the previous equations. Equation (6) will allow us to check our values for wavelength that are found experimentally in this lab. Equation (8) on the other hand will allow us to calculate the ground state energy for the Hydrogen atom once we know the wavelength of light and principle quantum numbers for a specific color.

3 Experiment

This experiment was conducted with the use of a spectroscope and hydrogen as the source of light. The experimental set up is illustrated as follows in Figure 1.

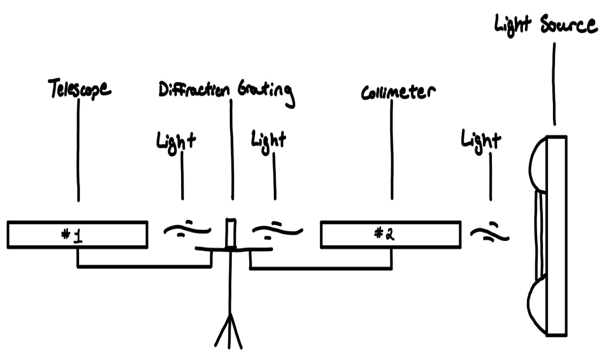

Figure 1: Experimental Set Up

The light first travels through the collimeter, then a diffraction grating, then out a telescope to where the observer can see the light. By rotating the telescope, we can observe different bright fringes of light and be able to record the angle between the two separate fringes.

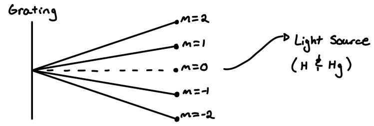

Figure 2: Maxima

Figure 2 depicts the view from above for this experiment with different orders of maxima. By swiveling the telescope and using the angular scale in it we can measure the angle in between the maxima (Such as *m±1*) at different maxima and for different colors. Figure 3 shows the trigonometry of this double-slit experiment.

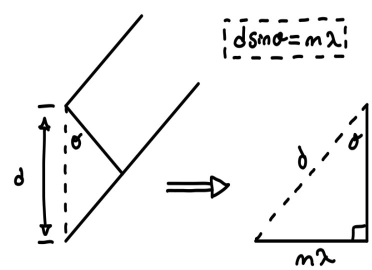

Figure 3: Trigonometry of Experiment

The double-slit interference equation as seen in Figure 3 that was used in this lab to calculate the angles in between maxima is [2]

$$d \sin\theta = m\lambda, \tag{9}$$

where the m is the order of maximum, d is the slit spacing, θ is the angle opposite the wavelength, and λ is the wavelength of the light in equation (9). For the angle θ, we are only interested in the angle between $m = 0$ and $m = 1$ since that is the angle that equation (9) uses. By physically being able to measure the angle between the maxima the only two unknown variables left are d and λ which are what we are trying to determine. Prior to observing a hydrogen spectrum, a mercury source was used as the light source to find the slit spacing *d*. According to the NIST atomic spectra database [1] the accepted wavelength for green light from a mercury source is known to be $\lambda = 546.075$ nm. For the $m \pm 1$ maxima, the angle between them was found to be $\frac{559}{30} \approx 18.6°$ where as the $m \pm 2$ maxima's angle in between was found to be 37.6° when mercury was being used as the light source. The slit spacing from these two maxima was found to be $d = 3.373 \pm 0.002$ µm.

4 Data

For the maxima of $m = \pm 1$, Table 1 has the data for the wavelengths of light. The θ value is the angle that can be seen in Figure 3. Hydrogen is abbreviated with an H.

Element	Color	θ in °	λ in nm
H	Purple	7.6	445.7
H	Green	8.5	499.0
H	Red	11.6	676.2

Table 1: $m \pm 1$ Maxima

Two other maxima were used to observe the wavelengths of light. The wavelengths of light at the $m \pm 2$ maxima can be seen in Table 2.

Element	Color	θ in °	λ in nm
H	Purple	15.3	445.5
H	Green	17.2	498.8
H	Red	23.5	672.7

Table 2: $m \pm 2$ Maxima

The maxima when $m = \pm 3$ did not all give a bright enough fringe for every color to get accurate measurements at this order. In short, the purple and green fringes can be disregarded thanks to the data rejection criterion. But for all intensive purposes the measured wavelengths at the third order maxima can be seen in Table 3.

Element	Color	θ in °	λ in nm
H	Purple	27.9	525.1
H	Green	30.6	572.3
H	Red	37.2	678.8

Table 3: $m \pm 3$ Maxima

First omitting the values of $m = \pm 3$ for the green and purple light, then taking the averages of the wavelengths and finding their standard deviation of the mean with the use of excel, Table 4 is created to summarize the results from all three maxima.

Color of Light	Average Wavelength λ nm
Purple	445.3 ± 0.1
Green	498.9 ± 0.1
Red	675.9 ± 1.8

Table 4: Summary of Results

The value for the red wavelength in Table 4 has all three maxima measurements where as the green and purple light both only consist of the second maxima measurements. This smaller data sample size actually improves the error in the wavelength where as when all three maxima were used to do the same calculations the errors were much greater.

5 Discussion

The average wavelengths found in Table 4 can now be compared to the theoretical wavelengths by knowing principal quantum numbers for each color. By doing the calculations with equation (6) and then comparing them to the average wavelengths it was found that red was found in the $n_i = 3$ level where green and purple were found in the 4^{th} and 5^{th} levels respectively. Table 5 has all of the average measured wavelengths found in this lab compared to their theoretical wavelengths.

Color	n_i	Measured λ nm	Theoretical λ nm
Purple	5	445.3 ± 0.1	433.9
Green	4	498.9 ± 0.1	486.0
Red	3	675.9 ± 1.8	656.1

Table 5: Measured vs. Theoretical λ Values

The results from Table 5 show that all three colors measurement's do not lie within the range of their theoretical values. The red light has the smallest deviation from the theoretical value where as the green and purple wavelengths have a greater deviation. With the measured wavelengths of light we can use them to calculate the constant E_0 which is equal to 13.6 eV or $2.176 * 10^{-18}$ J. Using the same n_i levels and corresponding colors from Table 5 equation (8) is used to calculate the measured E_0 value from this experiments data. Taking $h = 6.626 * 10^{-34}$ J s and $c = 3.0 * 10^{8}$ m s^{-1} [3], Table 6 summarizes the results from this experiment.

Color	n_i	Measured λ nm	Measured E_0 eV
Purple	5	445.3 ± 0.1	13.271 ± 0.003
Green	4	498.9 ± 0.1	13.265 ± 0.003
Red	3	675.9 ± 1.8	13.577 ± 0.001

Table 6: Measured E_0 Values

The results from Table 6 are not exact with accepted value of $E_0 = 13.6$ eV but are relatively close. It was once again found that the red light produced the most accurate results in the experiment.

6 Conclusion

From all of the colors that were observed in this lab the red light provided the most accurate results in comparison to the accepted values that were being measured in this lab. The darker colors (Green and Purple) were harder to see at larger maxima due to their intensity being less at these maxima. The colors with the smaller wavelengths have greater energy than the colors with the longer wavelengths. As the maxima increased in the double-slit interference set up, less and less photons of light were hitting the screen so it made it harder for an observer to see the colors at these maxima. This difficulty in being able to see the strips of light caused a lot of the discrepancies in this lab. If the light is not easily seen it cannot be accurately lined up with the telescopes to make measurements. There is also a possibility that the angles that were being read off of the angular scale might have been read off slightly incorrect. In order to improve the results that come from this experiment a darker room is needed to make the intensity greater so the measurements can be easier to take.

References

[1] Database, N. A., Database, A. S. (2018, January 1). NIST. Retrieved from NIST Atomic Spectra Data Base.

[2] Knight, Randall Dewey. Physics for scientists and engineers: a strategic approach: with modern physics. Pearson, 2013.

[3] Thornton, Stephen T. Modern Physics for Scientists and Engineers. Boston: Brooks/ Cole, 2013.